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Form Factors from Vertex Operators and Correlation Functions at $q = 1$

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We present a new application of affine Lie algebras to massive quantum field theory in 2 dimensions, by investigating the $q \to 1$ limit of the q-deformed affine $\widehat{sl(2)}$ symmetry of the sine-Gordon theory, this limit occurring at the free fermion point. We describe how radial quantization leads to a quasi-chiral factorization of the space of fields. The conserved charges which generate the affine Lie algebra split into two independent affine algebras on this factorized space, each with level 1, in the anti-periodic sector. The space of fields in the anti-periodic sector can be organized using level-1 highest weight representations, if one supplements the $\widehat{sl(2)}$ algebra with the usual local integrals of motion. Using the integrals of motion, a momentum space bosonization involving vertex operators is formulated. This leads to a new way of computing form-factors, as vacuum expectation values in momentum space. The problem of non-trivial correlation functions in this model is also discussed; in particular it is shown how space-time translational anomalies which arise in radial quantization can be used to compute the short distance expansion of some simple correlation functions.

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1. Introduction

It is well known that massive integrable quantum field theories in two dimensions are characterized by an infinite number of local integrals of motion P_n which all commute. Under Lorentz transformations these integrals of motion are characterized by their integer Lorentz spin

$$[L, P_n] = n \ P_n$$

where L is the Lorentz boost operator. For example, in the sine-Gordon theory, there exist P_n's for all odd integer n. The local integrals of motion are important for establishing integrability and consequently the factorizability of the S-matrix. They also play a central role in the quantum inverse scattering method.

Since the local integrals of motion satisfy an absolutely uninteresting infinite abelian algebra, they provide only limited information on the S-matrix, and have not provided any useful Ward identities for correlation functions. More recently, attention has turned to the existence of infinite non-abelian symmetries. Generally, these correspond to q-deformed affine Lie algebras and Yangians. The extent to which these quantum affine symmetries characterize the dynamical properties of the models remains an open question. The main dynamical properties one is interested in are the S-matrix, the form factors and the correlation functions. The fact that the S-matrix is almost completely characterized by its quantum affine symmetry suggests it is worthwhile to understand the consequences of this symmetry for the form factors and correlation functions.

We will consider the sine-Gordon theory, characterized by the lagrangian

$$S = \frac{1}{4\pi} \int d^2 z \left(\partial_z \phi \partial_{\overline{z}} \phi + 4\lambda \cos(\widehat{\beta}\phi) \right). \tag{1.1}$$

In [1][2], explicit conserved currents were constructed for the finite number of simple roots of the $q - \widehat{sl(2)}$ affine Lie algebra, where $q = \exp(-2\pi i/\widehat{\beta}^2)$. The main features of these currents are that they are non-local and possess fractional Lorentz spin $\pm(2/\widehat{\beta}^2 - 1)$. All of the Hopf algebraic properties of the quantum affine symmetry are a consequence of this non-locality, in particular, q^2 is a braiding phase. The non-locality of these conserved charges is precisely what makes it difficult to use them to derive Ward identities for example. To simplify the problem, consider the 'reflectionless' points $\widehat{\beta}^2 = 2/(N+1)$. Here, $q^2 = 1$, and the Lorentz spin of the charges is N. One is thus led to suspect the existence of an undeformed affine Lie algebra symmetry $\widehat{sl(2)}$ at all the reflectionless points, however this does not follow from results in [1] due to the delicacy of the $q^2 \to 1$ limit. In this talk

we will consider only the case of $N=1$, which occurs at the free fermion point. Indeed, one can construct explicitly all of the generators for the $\widehat{sl(2)}$ symmetry in this case. The aim of this work is to understand the extent to which the affine symmetry characterizes the dynamical properties of the model, and to develop some new structures that may be amenable to q-deformation. The deformation of the results presented here away from the free fermion point still remains a difficult open problem. Some aspects of the quantum affine symmetry at general reflectionless points is studied in [3]

Most of the work presented here is based on the papers[4][5]. Only the last section contains previously unpublished work.

2. $\widehat{sl(2)}$ Symmetry of Massive Dirac Fermions

In this section we describe the conserved currents for the affine $\widehat{sl(2)}$ symmetry at the free fermion point. It is well known that at $\widehat{\beta}=1$, the sine-Gordon theory is equivalent to a massive free field theory of charged fermions. Introducing the Dirac spinors $\Psi_\pm = \begin{pmatrix} \overline{\psi}_\pm \\ \psi_\pm \end{pmatrix}$ of $U(1)$ charge ± 1, the equations of motion are

$$\partial_z \overline{\psi}_\pm = i\widehat{\mathrm{m}}\psi_\pm, \qquad \partial_{\overline{z}}\psi_\pm = -i\widehat{\mathrm{m}}\overline{\psi}_\pm. \tag{2.1}$$

We have continued to Euclidean space, and $z, \overline{z}$ are the usual Euclidean light-cone coordinates: $z=(t+ix)/2, \qquad \overline{z}=(t-ix)/2$.

Generally, the conserved quantities Q follow from conserved currents J_μ:

$$\partial_{\overline{z}} J_z + \partial_z J_{\overline{z}} = 0; \qquad Q = \int \frac{dz}{2\pi i}\ J_z \ - \ \int \frac{d\overline{z}}{2\pi i}\ J_{\overline{z}}. \tag{2.2}$$

Define the inner product of two spinors $A = \begin{pmatrix} \overline{a} \\ a \end{pmatrix}$, $B = \begin{pmatrix} \overline{b} \\ b \end{pmatrix}$ as

$$(A,B) = \frac{1}{4\pi}\left(\int dz\ a\, b \ + \ \int d\overline{z}\overline{a}\,\overline{b} \right). \tag{2.3}$$

Using the equations of motion (2.2), one can verify that the following charges are conserved:

$$\begin{aligned} Q^\pm_{-n} &= \frac{1}{2}\,(\Psi_\pm, \partial_z^n \Psi_\pm), \qquad Q^\pm_n = \frac{1}{2}\,(\Psi_\pm, \partial_{\overline{z}}^n \Psi_\pm) \qquad n \text{ odd} \\ \alpha_{-n} &=: (\Psi_+, \partial_z^n \Psi_-) :, \qquad \alpha_n =: (\Psi_+, \partial_{\overline{z}}^n \Psi_-) : \end{aligned} \tag{2.4}$$

where $n \geq 0$.

Define

$$P_n = \alpha_n, \quad n \text{ odd}; \qquad T_n = \alpha_n, \quad n \text{ even}.$$

Then one can show that the charges obey the following commutation relations:

$$[P_n, P_m] = 0 \tag{2.5a}$$

$$[P_n, T_m] = \left[P_n, Q_m^{\pm}\right] = 0 \tag{2.5b}$$

$$[T_n, T_m] = 0 \tag{2.5c}$$

$$\left[T_n, Q_m^{\pm}\right] = \pm 2 \, \frac{\widehat{\mathrm{m}}^{|n|+|m|}}{\widehat{\mathrm{m}}^{|n+m|}} \, Q_{n+m}^{\pm} \tag{2.5d}$$

$$\left[Q_n^{+}, Q_m^{-}\right] = \frac{\widehat{\mathrm{m}}^{|n|+|m|}}{\widehat{\mathrm{m}}^{|n+m|}} \, T_{n+m} \tag{2.5e}$$

The conserved charges have integer Lorentz spin:

$$[L, \alpha_n] = -n \, \alpha_n, \qquad \left[L, Q_n^{\pm}\right] = -n \, Q_n^{\pm}. \tag{2.6}$$

The P_n are the usual local integrals of motion at odd Lorentz spin, where the translation operators are $P_z = P_{-1}, \;\; P_{\overline{z}} = P_1$. The remaining generators $T_n, \; Q_n^{\pm}$ satisfy the *twisted* $\widehat{sl(2)}$ algebra at level 0, and furthermore commute with the P_n. The twist is to be expected here since in the free fermion theory only the U(1) charge is Lorentz spinless. The twist breaks the sl(2) subalgebra of zero modes that is familiar in current algebra to U(1).

It is interesting to consider the conformal limit of the above charges. As $\widehat{\mathrm{m}} \to 0$, the charges Q_{-n} for $n \geq 0$ become purely left-moving whereas the Q_n become right-moving. Thus, in the conformal limit one recovers two decoupled Borel subalgebras of $\widehat{sl(2)}$.

The lowest conserved charges $Q_1^{\pm}, \; Q_{-1}^{\pm}$ can be derived from the results in [1] using standard bosonization formulas, in fact this is how we originally found them. See also [6][7]

3. Representation Theory

In this section we consider the representations of $\widehat{sl(2)}$ that are realized in the model. There are two vector spaces of interest: the space of particles and the space of fields. The space of particles diagonalizes the translation operators, which on one particle states have the eigenvalues $P_z = \widehat{\mathrm{m}} e^{\theta}, \;\; P_{\overline{z}} = \widehat{\mathrm{m}} e^{-\theta}$. The space of particles is then:

$$\mathcal{H}_P = \{\oplus |\theta_1 \cdots \theta_n\rangle\}.$$

On the doublet of one-particle states, the conserved charges have the following loop algebra (level 0) representation:

$$\alpha_n = \widehat{\mathrm{m}}^{|n|} u^{-n} \begin{pmatrix} 1 & 0 \\ 0 & (-1)^{n+1} \end{pmatrix}, \quad Q_n^+ = \widehat{\mathrm{m}}^{|n|} u^{-n} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad Q_n^- = \widehat{\mathrm{m}}^{|n|} u^{-n} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{3.1}$$

The trivial comultiplication induces a representation of $\widehat{sl(2)}$ on the tensor product space $\mathcal{H}_P$. Thus $\mathcal{H}_P$ is a direct sum of finite dimensional level 0 representations of $\widehat{sl(2)}$.

Consider now the space of fields. Let $\Phi_i(z, \overline{z})$ denote a complete basis in the space of fields and let $|\Phi_i >= \Phi_i(0)|0>$. The space of fields is then defined as

$$\mathcal{H}_F = \{\oplus_i |\Phi_i >\}. \tag{3.2}$$

The space of fields of course does not diagonalize the translation operators $P_z, P_{\overline{z}}$, but rather diagonalizes the Lorentz boost operator L. The represions of $\widehat{sl(2)}$ on the space of fields are much more interesting than on the space of particles. An explicit construction of the space of fields can be obtained via radial quantization, since it is this quantization scheme that yields states that diagonalize L. In this way one can show that there exists a quasi-chiral splitting in the space of fields:

$$\mathcal{H}_F = \mathcal{H}_F^L \otimes \mathcal{H}_F^R.$$

In the conformal limit, $\mathcal{H}_F^{L,R}$ are the left and right moving chiral sectors; however we emphasize that radial quantization provides a meaning to the quasi-chiral splitting in the massive theory. Furthermore, as I will describe, the conserved charges split into quasi left and right pieces, and the spaces $\mathcal{H}_F^{L,R}$ turn out to be infinite dimensional level 1 representations of $\widehat{sl(2)}$.

We now describe the basic features of radial quantization that allows one to establish the above statements. In radial quantization, equal 'time' surfaces are taken to be circles surrounding the origin in the space-time plane. For the free fermion, one only needs to solve the Dirac equation in polar coordinates to obtain the expansions of the fields appropriate to radial quantization. One finds

$$\begin{pmatrix} \overline{\psi}_\pm \\ \psi_\pm \end{pmatrix} = \sum_\omega b_\omega^\pm \ \Psi^{(a)}_{-\omega-1/2} \ + \ \overline{b}_\omega^\pm \ \overline{\Psi}^{(a)}_{-\omega-1/2}. \tag{3.3}$$

The labels a, p represent anti-periodic versus periodic sectors. In the anti-periodic sector, $\omega \in Z$, whereas in the periodic sector $\omega \in Z + 1/2$. We will mainly be concerned with the anti-periodic sector, where one has:

$$
\begin{aligned}
\Psi^{(a)}_{-\omega-1/2} &= \Gamma(\tfrac{1}{2}-\omega)\ \widehat{\mathrm{m}}^{\omega+1/2}\ \begin{pmatrix} i\, e^{i(\frac{1}{2}-\omega)\varphi}\ I_{\frac{1}{2}-\omega}(\widehat{\mathrm{m}}r) \\ e^{-i(\omega+\frac{1}{2})\varphi}\ I_{-\omega-\frac{1}{2}}(\widehat{\mathrm{m}}r) \end{pmatrix} \\
\overline{\Psi}^{(a)}_{-\omega-1/2} &= \Gamma(\tfrac{1}{2}-\omega)\ \widehat{\mathrm{m}}^{\omega+1/2}\ \begin{pmatrix} e^{i(\frac{1}{2}+\omega)\varphi}\ I_{-\frac{1}{2}-\omega}(\widehat{\mathrm{m}}r) \\ -i\, e^{-i(\frac{1}{2}-\omega)\varphi}\ I_{\frac{1}{2}-\omega}(\widehat{\mathrm{m}}r) \end{pmatrix}.
\end{aligned}
\tag{3.4}
$$

Similar formulas apply to the periodic sector.

Upon quantization, one finds

$$
\{b^+_\omega, b^-_{\omega'}\} = \delta_{\omega,-\omega'}, \qquad \{\overline{b}^+_\omega, \overline{b}^-_{\omega'}\} = \delta_{\omega,-\omega'}, \qquad \{b_\omega, \overline{b}_{\omega'}\} = 0. \tag{3.5}
$$

Since the b operators commute with the $\overline{b}$ operators, the fermionic Fock space built out of the the fermion modes factorizes as explained above.

The fermionic Fock spaces constructed using the operators $b_\omega,\ \overline{b}_\omega$ can be identified with the space of fields. For example, in the periodic sector one finds

$$
\partial^n_z \psi_\pm(0)|0\rangle = n!\ b^\pm_{-n-\frac{1}{2}}|0\rangle, \qquad \partial^n_{\overline{z}} \overline{\psi}_\pm(0)|0\rangle = n!\ \overline{b}^\pm_{-n-\frac{1}{2}}|0\rangle. \tag{3.6}
$$

$$
J_z(0)|0\rangle = b^+_{-\frac{1}{2}}\ b^-_{-\frac{1}{2}}\ |0\rangle, \qquad J_{\overline{z}}(0)|0\rangle = \overline{b}^+_{-\frac{1}{2}}\ \overline{b}^-_{-\frac{1}{2}}\ |0\rangle, \tag{3.7}
$$

where J_μ is the U(1) current. The above formulas are identical to the conformal limit, however we emphasize that they are valid in the massive theory: they are a consequence of the $r \to 0$ properties of the Bessel functions in the expansion of the fields.

In the anti-periodic sector, due to the existence of the zero modes $b_0,\ \overline{b}_0$, there are doubly degenerate Ramond vacuua for each sector $|\pm 1/2>_L$ and $|\pm 1/2>_R$. One has the following identification with fields in the sine-Gordon theory:

$$
e^{\pm i\phi(0)/2}\ |0\rangle\ =\ (c\widehat{\mathrm{m}})^{1/4}\ \left(|\pm\tfrac{1}{2}\rangle_L \otimes |\mp\tfrac{1}{2}\rangle_R\right) \equiv (c\widehat{\mathrm{m}})^{1/4}\ |\pm\tfrac{1}{2}\rangle, \tag{3.8}
$$

where c is some constant. The quasi-chiral spaces of fields described above have the following explicit realization:

$$
\begin{aligned}
\mathcal{H}^L_{a\pm} &= \left\{ b^-_{-n_1} b^-_{-n_2} \cdots b^+_{-n'_1} b^+_{-n'_2} \cdots |\pm\tfrac{1}{2}\rangle_L \right\} \\
\mathcal{H}^R_{a\pm} &= \left\{ \overline{b}^-_{-n_1} \overline{b}^-_{-n_2} \cdots \overline{b}^+_{-n'_1} \overline{b}^+_{-n'_2} \cdots |\pm\tfrac{1}{2}\rangle_R \right\},
\end{aligned}
\tag{3.9}
$$

for $n, n' \geq 1$.

In the radial quantization the conserved charges split into quasi left and right pieces:

$$\begin{aligned} Q^{\pm}_n &= Q^{\pm,L}_n + Q^{\pm,R}_{-n} \\ \alpha_n &= \alpha^L_n + \alpha^R_{-n}. \end{aligned} \tag{3.10}$$

The Q^L (Q^R) pieces involve only the b ($\overline{b}$) operators. For instance,

$$Q^{\pm,L}_n = \widehat{\mathrm{m}}^{|n|+n} \sum_{\omega \in Z} \frac{\Gamma(\frac{1}{2}+\omega-n)}{\Gamma(\frac{1}{2}+\omega)} \, b^{\pm}_{n-\omega} b^{\pm}_{\omega}. \tag{3.11}$$

With the above expression for the conserved charges in terms of the fermion modes, one can study the representation of the charges on the space of fields. One finds that the split charges now satisfy a level 1 affine Lie algebra:

$$\begin{aligned} \left[P^L_n, P^L_m\right] &= n \ \widehat{\mathrm{m}}^{2|n|} \ \delta_{n,-m} \\ \left[P^L_n, T^L_m\right] &= \left[P^L_n, Q^{\pm,L}_m\right] = 0 \\ \left[T^L_n, T^L_m\right] &= n \ \widehat{\mathrm{m}}^{2|n|} \ \delta_{n,-m} \\ \left[T^L_n, Q^{\pm,L}_m\right] &= \pm 2 \ \frac{\widehat{\mathrm{m}}^{|n|+|m|}}{\widehat{\mathrm{m}}^{|n+m|}} \ Q^{\pm,L}_{n+m} \\ \left[Q^{+,L}_n, Q^{-,L}_m\right] &= \frac{\widehat{\mathrm{m}}^{|n|+|m|}}{\widehat{\mathrm{m}}^{|n+m|}} \ T^L_{n+m} \ + \ \frac{n}{2} \ \widehat{\mathrm{m}}^{2|n|} \ \delta_{n,-m}, \end{aligned} \tag{3.12}$$

and similarly for the right charges. The non-zero levels cancel in the sum $Q^L + Q^R$ so that the complete charges continue to satisfy a level 0 algebra.

Let us denote the P_n extension of the algebra $\widehat{sl(2)}$ defined in (3.12) as $\widehat{\widehat{sl(2)}}$. As we have seen, the symmetry algebra factorizes into $\widehat{\widehat{sl(2)}}_L \otimes \widehat{\widehat{sl(2)}}_R$. The algebra $\widehat{\widehat{sl(2)}}$ is a complete spectrum generating algebra for the anti-periodic sector, namely, the complete spectrum of quasi-chirally factorized fields can be obtained from infinite highest weight representations of $\widehat{\widehat{sl(2)}}$. To show this define modules

$$\begin{aligned} \widehat{\widehat{V}}{}^L_{\Lambda_0} &\equiv \left\{ Q^{\pm,L}_{-n_1} \ Q^{\pm,L}_{-n_2} \cdots T^L_{-n'_1} T^L_{-n'_2} \cdots P^L_{-n''_1} P^L_{-n''_2} \cdots |-\tfrac{1}{2}\rangle_L \right\} \\ \widehat{\widehat{V}}{}^L_{\Lambda_{\frac{1}{2}}} &\equiv \left\{ Q^{\pm,L}_{-n_1} \ Q^{\pm,L}_{-n_2} \cdots T^L_{-n'_1} T^L_{-n'_2} \cdots P^L_{-n''_1} P^L_{-n''_2} \cdots |+\tfrac{1}{2}\rangle_L \right\}, \end{aligned} \tag{3.13}$$

for $n, n', n'' \geq 1$. Then, using characters one can show that

$$\mathcal{H}^L_a = \widehat{\widehat{V}}{}^L_{\Lambda_0} \oplus \widehat{\widehat{V}}{}^L_{\Lambda_{\frac{1}{2}}}. \tag{3.14}$$

4. Particle-Field Duality and Form-Factors from Vertex Operators

Form factors are matrix elements of fields in the space of states $\mathcal{H}_P$. The basic form factors

$$ {}^{\epsilon_1\cdots\epsilon_n}\langle\theta_1\cdots\theta_n|\Phi(0)|0>, $$

from which the more general matrix elements may be obtained by crossing symmetry, are inner products of states in $\mathcal{H}_F$ with states in $\mathcal{H}_P^*$. (Here, the indices ϵ_i are isotopic.) The completeness relation in the space of particles,

$$ 1=\sum_{\vec{\theta}}|\overrightarrow{\theta}\rangle\langle\overleftarrow{\theta}|=\sum_{n=0}^{\infty}\frac{1}{n!}\sum_{\{\epsilon_i\}}\int d\theta_1\cdots d\theta_n\ |\theta_1,\cdots,\theta_n\rangle_{\epsilon_1\cdots\epsilon_n}{}^{\epsilon_n\cdots\epsilon_1}\langle\theta_n,\cdots,\theta_1|. \qquad (4.1) $$

allows us to map states in $\mathcal{H}_F$ to states in $\mathcal{H}_P$, i.e. to view $|\Phi\rangle\in\mathcal{H}_P$:

$$ |\Phi_i\rangle=\sum_{\vec{\theta}}|\overrightarrow{\theta}\rangle\langle\overleftarrow{\theta}|\Phi_i\rangle. \qquad (4.2) $$

The intuitive simplicity of the space $\mathcal{H}_P$ is responsible for this conventional way of thinking about form factors.

We give now a dual description of the same form factors. Let us suppose that one can define a dual to the space of fields $\mathcal{H}_F^*$ with inner product and completeness relation:

$$ \begin{aligned} \langle\Phi^i|\Phi_j\rangle&=\delta^i_j\\ 1&=\sum_i|\Phi_i\rangle\langle\Phi^i|. \end{aligned} \qquad (4.3) $$

Then one can map a state $|\vec{\theta}\rangle\in\mathcal{H}_P$ into $\mathcal{H}_F$. The dual statement is

$$ {}^{\epsilon_1\cdots\epsilon_n}\langle\theta_1\cdots\theta_n|=\sum_{\Phi_i\in\mathcal{F}}{}^{\epsilon_1\cdots\epsilon_n}\langle\theta_1\cdots\theta_n|\Phi_i\rangle\langle\Phi^i|\qquad\in\mathcal{H}_F^*. \qquad (4.4) $$

In order to work the above simple remarks into an efficient means of computing form factors, one must work explicitly with the space $\mathcal{H}_F$. Radial quantization provides the necessary structures (4.3). In order to use these ideas to compute form factors, we need to introduce the notion of vertex operators. The formula (4.4) implies that one can map states in $\mathcal{H}_P^*$ to states in $\mathcal{H}_F^*$. We call this map the 'particle-field map'. We construct this map explicitly by defining vertex operators $V^\epsilon(\theta)$ as follows:

$$ {}^{\epsilon_1\cdots\epsilon_n}\langle\theta_1\cdots\theta_n|=\langle\Omega|\ V^{\epsilon_1}(\theta_1)\cdots V^{\epsilon_n}(\theta_n)\quad\in\mathcal{H}_F^* \qquad (4.5) $$

where $\langle\Omega|$ is a fixed 'vacuum' state, which will be characterized completely below. The vertex operators are distinguished from the Faddeev-Zamolodchikov operators $Z(\theta)$ since they act on completely different spaces. However the basic algebraic relations satisfied by the Z operators continue to be satisfied by the V operators. The vertex operators $V^\epsilon(\theta)$ operate in the space $\mathcal{H}_F$:

$$V^\epsilon(\theta): \qquad \mathcal{H}_F \to \mathcal{H}_F. \tag{4.6}$$

In the sequel we will describe how to construct these vertex operators explicitly using radial quantization. Once the vertex operators are constructed, the form factors ${}^{\epsilon_1\cdots\epsilon_n}\langle\theta_1\cdots\theta_n|\Phi_i\rangle$ are computed directly in the discrete space $\mathcal{H}_F$ using (4.5).

The vertex operators can be constructed explicitly using the radial modes introduced in the last section. Let

$$u = e^\theta,$$

and define

$$\begin{aligned} b^\pm(u) &= \pm i \sum_{\omega\in Z} \Gamma(\tfrac{1}{2}-\omega)\,\widehat{\mathrm{m}}^\omega\; b^\pm_\omega\; u^\omega \\ \overline{b}^\pm(u) &= \pm \sum_{\omega\in Z} \Gamma(\tfrac{1}{2}-\omega)\,\widehat{\mathrm{m}}^\omega\; \overline{b}^\pm_\omega\; u^{-\omega}. \end{aligned} \tag{4.7}$$

Then the vertex operators are

$$\begin{aligned} \langle\Omega| &= \langle\tfrac{1}{2}| + \langle-\tfrac{1}{2}| \\ V^\epsilon(\theta) &= \frac{1}{\sqrt{2\pi^2 i}}\left(b^\epsilon(e^{-i\pi}u) + \overline{b}^\epsilon(e^{-i\pi}u)\right). \end{aligned} \tag{4.8}$$

The normalization of the vertex operators was chosen to satisfy the residue property

$$V^+(\theta)V^-(\theta+\beta+i\pi) \;\sim\; \frac{1}{i\pi\,\beta}, \tag{4.9}$$

which leads to the proper residue axiom for the multiparticle form factors. From (3.8), one sees that the choice (4.8) for $\langle\Omega|$ is equivalent to the following vacuum expectation values:

$$\langle 0|\; e^{\pm i\phi(0)/2}\; |0\rangle = (c\widehat{\mathrm{m}})^{1/4} \tag{4.10}$$

One can use the above construction to compute the form factors of the fields $\exp(\pm i\phi/2)$. For these fields, all of the form factors with a $U(1)$ neutral combination

of an even number of particles is non-zero. The result is

$$
\begin{aligned}
&{}^{+++\ldots---\ldots}\langle\theta_1,\theta_2,\cdots,\theta_{2n}|\; e^{\pm i\phi(0)/2}|0\rangle\\
&= (c\widehat{\mathrm{m}})^{1/4}\; \langle\mp\tfrac{1}{2}|V^+(u_1)\cdots V^+(u_n)V^-(u_{n+1})\cdots V^-(u_{2n})|\pm\tfrac{1}{2}\rangle\\
&= (c\widehat{\mathrm{m}})^{1/4}\frac{(\pm 1)^n}{(i\pi)^n}(-1)^{n(n-1)/2}\sqrt{u_1\cdots u_{2n}}\left(\prod_{i=1}^{n}\left(\frac{u_{i+n}}{u_i}\right)^{\pm 1/2}\right)\left(\prod_{i<j\le n}(u_i-u_j)\right)\\
&\quad\times\left(\prod_{n+1\le i<j}(u_i-u_j)\right)\left(\prod_{r=1}^{n}\prod_{s=n+1}^{2n}\frac{1}{u_r+u_s}\right).
\end{aligned}
\tag{4.11}
$$

The above computation can be done using the Wick theorem with the 2-point functions

$$
\begin{aligned}
{}_L\langle-\tfrac{1}{2}|\; b^+(u)\, b^-(u')\;|+\tfrac{1}{2}\rangle_L &= {}_R\langle+\tfrac{1}{2}|\;\overline{b}^+(u)\,\overline{b}^-(u')\;|-\tfrac{1}{2}\rangle_R = \pi\frac{u'}{u+u'}\\
{}_L\langle+\tfrac{1}{2}|\; b^+(u)\, b^-(u')\;|-\tfrac{1}{2}\rangle_L &= {}_R\langle-\tfrac{1}{2}|\;\overline{b}^+(u)\,\overline{b}^-(u')\;|+\tfrac{1}{2}\rangle_R = -\pi\frac{u}{u+u'}.
\end{aligned}
\tag{4.12}
$$

However the computation is more easily done using the bosonization techniques of the next section. After some algebraic manipulation, one can see that these expressions agree with the known results, though they were originally computed using rather different methods[8][9].

5. Bosonization in Momentum Space

In the massive theory one can use the constants of motion to formulate an exact bosonization. In the anti-periodic sector, since $\alpha_n^{L,R}$ satisfy two separate Heisenberg algebras, they can be used to construct a bosonization. Define

$$
\rho_n = \widehat{\mathrm{m}}^{-|n|}\alpha_n^L, \qquad \overline{\rho}_n = \widehat{\mathrm{m}}^{-|n|}\alpha_n^R,
$$

and define the momentum space fields (recall $u = e^\theta$):

$$
\begin{aligned}
-i\rho(u) &= \sum_{n\neq 0}\; \rho_n\;\frac{u^n}{n} + \rho_0\log(u) - \widetilde{\rho}_0\\
-i\overline{\rho}(u) &= \sum_{n\neq 0}\; \overline{\rho}_n\;\frac{u^{-n}}{n} - \overline{\rho}_0\log(u) - \widetilde{\overline{\rho}}_0,
\end{aligned}
\tag{5.1}
$$

where one has $[\rho_0,\widetilde{\rho}_0] = [\overline{\rho}_0,\widetilde{\overline{\rho}}_0] = 1$. We further define an auxiliary vacuum satisfying

$$
\alpha_n^L|\emptyset\rangle = \alpha_n^R|\emptyset\rangle = 0, \quad n\ge 0; \qquad \widetilde{\alpha}_0^L|\emptyset\rangle\;,\;\; \widetilde{\alpha}_0^R|\emptyset\rangle \neq 0. \tag{5.2}
$$

This vacuum $|\emptyset\rangle$ is not to be confused with the physical vacuum $|0\rangle$ which resides in the periodic sector. One has the following expectation values:

$$
\begin{aligned}
\langle\emptyset|\ \rho(u)\ \rho(u')\ |\emptyset\rangle &= -\log(1/u - 1/u') \\
\langle\emptyset|\ \overline{\rho}(u)\ \overline{\rho}(u')\ |\emptyset\rangle &= -\log(u - u')
\end{aligned}
\tag{5.3}
$$

$$
\begin{aligned}
\langle\emptyset| \prod_i\ e^{i\alpha_i \rho(u_i)}\ |\emptyset\rangle &= \prod_{i<j} (1/u_i - 1/u_j)^{\alpha_i\alpha_j} \\
\langle\emptyset| \prod_i\ e^{i\alpha_i \overline{\rho}(u_i)}\ |\emptyset\rangle &= \prod_{i<j} (u_i - u_j)^{\alpha_i\alpha_j}\ .
\end{aligned}
\tag{5.4}
$$

The bosonized expressions for the operators $b^{\pm}(u), \overline{b}^{\pm}(u)$ and the states $|\pm\frac{1}{2}\rangle$ follow from the basic commutation relations

$$
\begin{aligned}
\left[\alpha_n^L, b^{\pm}(u)\right] &= (\pm 1)^{n+1} \widehat{\mathrm{m}}^{|n|} u^{-n}\ b^{\pm}(u) \\
\left[\alpha_n^R, \overline{b}^{\pm}(u)\right] &= (\pm 1)^{n+1} \widehat{\mathrm{m}}^{|n|} u^{n}\ \overline{b}^{\pm}(u)
\end{aligned}
\tag{5.5}
$$

and the 2-point functions (4.12). The commutation relations (5.5) are fundamental in the sense that they describe how the conserved charges are represented on asymptotic multiparticle states. One finds

$$
\sqrt{\frac{\pm u}{\pi}}\ b^{\pm}(u) =: e^{\pm i\rho(\pm u)} :, \qquad \frac{\pm 1}{\sqrt{\pm\pi u}}\ \overline{b}^{\pm}(u) =: e^{\pm i\overline{\rho}(\pm u)} : \tag{5.6}
$$

where $-u = e^{-i\pi} u$, and

$$
\begin{aligned}
|\pm\tfrac{1}{2}\rangle_L &= : e^{\pm i\rho(\infty)/2} : |\emptyset\rangle_L, & |\pm\tfrac{1}{2}\rangle_R &= : e^{\pm i\overline{\rho}(0)/2} : |\emptyset\rangle_R \\
{}_L\langle\pm\tfrac{1}{2}| &= \lim_{u\to 0}\ u^{-1/4}\ \langle\emptyset| : e^{\pm i\rho(u)/2} :, & {}_R\langle\pm\tfrac{1}{2}| &= \lim_{u\to\infty}\ u^{1/4}\ \langle\emptyset| : e^{\pm i\overline{\rho}(u)/2} : .
\end{aligned}
\tag{5.7}
$$

One can easily check that this construction reproduces the form factors (4.11).

6. Differential Equations for Correlation Functions

The problem of correlation functions in massive integrable models is notoriously difficult and generally unsolved. Even here in the free fermion theory correlators of the fields $\exp(i\alpha\phi)$ for general α are rather complicated. For $\alpha = \pm 1/2$ these correlation functions are related to squares of Ising correlators[10][8]. From the celebrated Ising results of Wu et. al. [11] one can obtain differential equations for these special sine-Gordon correlators.

In [12] differential equations for derived for arbitrary α in the following way. Consider the two point function

$$< 0|e^{i\alpha\phi(z,\overline{z})}\ e^{i\alpha'\phi(0)}|0 > . \tag{6.1}$$

Inserting the resolution of the identity (4.1) between the fields and using the form-factors, one finds that this correlator can be written as a Fredholm determinant:

$$< 0|e^{i\alpha\phi(z,\overline{z})}\ e^{i\alpha'\phi(0)}|0 >= Det(1+\mathbf{K}), \tag{6.2}$$

where $\mathbf{K}$ is a 2 by 2 matrix of integral operators:

$$\mathbf{K}(\mathbf{u},\mathbf{v}) = \begin{pmatrix} 0 & \frac{e(u)\widehat{e}(v)}{u+v} \\ \frac{\widehat{e}(u)e(v)}{u+v} & 0 \end{pmatrix} \tag{6.3}$$

where

$$\begin{aligned} e(u) &= \left(\frac{\sin(\pi\alpha)}{\pi}\right)^{1/2} u^{(1+\alpha'-\alpha)/2} \exp\left[-\tfrac{1}{2}m(zu+\overline{z}u^{-1})\right] \\ \widehat{e}(u) &= \left(\frac{\sin(\pi\alpha')}{\pi}\right)^{1/2} u^{(1+\alpha-\alpha')/2} \exp\left[-\tfrac{1}{2}m(zu+\overline{z}u^{-1})\right] \end{aligned} \tag{6.4}$$

.

Using a generalization of the techniques developed in [13][14] one can show that the above two point functions are parameterized in terms of a solution η of the sinh-Gordon equation[12]. Let

$$\Sigma(z,\overline{z}) = \log < 0|e^{i\alpha\phi(z,\overline{z})}\ e^{i\alpha'\phi(0)}|0 > .$$

Then

$$\partial_z\partial_{\overline{z}}\Sigma = \frac{m^2}{2}(1-\cosh 2\eta)$$

$$\partial_z\partial_{\overline{z}}\eta = \frac{m^2}{2}\sinh 2\eta.$$

Note that the above differential equations are do not depend on α, α', though the correlation function itself certainly does. This implies that the α, α' dependence must come in as a boundary condition for the solution of the differential equation.

An important question is to understand whether the above differential equations can somehow be obtained from the affine $\widehat{sl(2)}$ symmetry. On the one hand, the above differential equations are known to be part of the differential equations of the $\widehat{sl(2)}$ hierarchy. However we know of no concrete connection between the later affine structure and the genuine affine symmetry of the quantum field theory. It would be very interesting to relate these two structures since this may provide some ideas on how to q-deform the above results on correlation functions, though this is rather speculative.

7. Translation Anomaly and Short Distance Expansion

The infinite integral representation (Fredholm determinant) that we obtained in the last section for the 2 point functions using the resolution of the identity and the form factors is a large distance expansion. The symmetry $\widehat{sl(2)}$ does not easily relate various terms in this form factor sum since it commutes with particle number. However it seems more likely that the $\widehat{sl(2)}$ symmetry could relate various terms in the short distance expansion. One reason for believing this is that the short distance expansion is just conformal perturbation theory, and the quantum affine charges were originally derived in this framework, however we have very little that is concrete to add to this speculation.

We would like to show how one can use the infinite Heisenberg algebras described above to recover the short distance expansion of a simple correlator. In the radial quantization, the integrals of motion split into chiral pieces each satisfying infinite Heisenberg algebras:

$$\left[\alpha_n^L, \alpha_m^L\right] = n\ \widehat{\mathrm{m}}^{2|n|}\ \delta_{n,-m}, \qquad \left[\alpha_n^R, \alpha_m^R\right] = n\ \widehat{\mathrm{m}}^{2|n|}\ \delta_{n,-m}. \tag{7.1}$$

For the translation operators, this implies

$$P_z = \alpha_{-1}^L + \alpha_1^R, \qquad P_{\overline{z}} = \alpha_1^L + \alpha_{-1}^R.$$

Of course the translation operators commute: $[P_z, P_{\overline{z}}] = 0$, however there are anomalies in the split pieces: $[\alpha_1^L, \alpha_{-1}^L] = [\alpha_1^R, \alpha_{-1}^R] = \widehat{\mathrm{m}}^2$. The idea is the use the non-manifest translation invariance in radial quantization to our advantage.

We will consider the correlation function:

$$\langle \tfrac{1}{2}|\psi_+(z,\overline{z})\psi_-(w,\overline{w})| - \tfrac{1}{2}\rangle = \frac{1}{z-w}\sqrt{\frac{w}{z}} + \mathcal{O}(\widehat{\mathrm{m}}^2) + ... \tag{7.2}$$

In conformal perturbation theory all of the higher order terms in powers of m have complicated integral representations. Namely,

$$< -\tfrac{1}{2}|\psi_+(z,\overline{z})\psi_-(w,\overline{w})|\tfrac{1}{2} >= \sum_{n=0}^{\infty}\left(\frac{\widehat{\mathrm{m}}}{\pi}\right)^{2n}\frac{1}{(2n)!}\binom{2n}{n}\int d^2z_1\cdots d^2z_{2n}\, C(z,\overline{z},w,\overline{w};z_1,\cdots z_{2n}), \tag{7.3}$$

where

$$C = \sqrt{\frac{z}{w}}\frac{1}{z-w}\frac{\left[\prod_{i,j=1,..n,i\neq j}|z_i-z_j|^2\right]\left[\prod_{i,j=n+1...2n,i\neq j}|z_i-z_j|^2\right]}{\left[\prod_{r=1}^{n}\prod_{s=n+1}^{2n}|z_r-z_s|^{-2}\right]} \times\left[\prod_{i=1}^{n}\frac{z_i-z}{z_i-w}\frac{z_{i+n}-\omega}{z_{i+n}-z}\frac{|z_i|}{|z_{i+n}|}\right]. \tag{7.4}$$

We will give a simple operator construction of all of these higher order terms.

One starts from the relation

$$\psi(z,\overline{z}) = e^{z\alpha_{-1}+\overline{z}\alpha_1}\ \psi(0)\ e^{-z\alpha_{-1}-\overline{z}\alpha_1}. \tag{7.5}$$

As we have seen, at $r=0$, we are very close to conformal field theory. Let us take the expression:

$$\psi(0) = \lim_{\epsilon,\overline{\epsilon}\to 0} \sum_\omega b_\omega \epsilon^{-\omega-1/2} + \frac{\widehat{\mathrm{m}}}{\frac{1}{2}-\omega}\overline{b}_\omega \overline{\epsilon}^{\frac{1}{2}-\omega}. \tag{7.6}$$

The first term above is just as in conformal field theory; the second term can be obtained from first order perturbation theory. Using

$$e^A\psi e^{-A} = \psi + [A,\psi] + \tfrac{1}{2}[A,[A,\psi]] + ... \tag{7.7}$$

and

$$[\alpha^L_{-1}, b_\omega] = (\tfrac{1}{2}-\omega)b_{\omega-1}, \qquad [\alpha^L_1, b_\omega] = -\frac{\widehat{\mathrm{m}}^2}{\frac{1}{2}+\omega}b_{\omega+1}$$

$$[\alpha^R_{-1}, \overline{b}_\omega] = (\tfrac{1}{2}-\omega)\overline{b}_{\omega-1}, \qquad [\alpha^R_1, \overline{b}_\omega] = -\frac{\widehat{\mathrm{m}}^2}{\frac{1}{2}+\omega}\overline{b}_{\omega+1}$$

one obtains

$$\psi(z,\overline{z}) = \sum_{\omega\in Z} b_\omega f_\omega(z,\overline{z}) + \overline{b}_\omega \overline{f}_\omega(z,\overline{z}). \tag{7.8}$$

The functions $f_\omega, \overline{f}_\omega$ are easily computable order by order in $\widehat{\mathrm{m}}$ from (7.5)(7.7). One obtains

$$f_\omega = z^{-\omega-\frac{1}{2}}\left(\sum_{n=0}^{\infty}(-1)^n\frac{(\widehat{\mathrm{m}}^2 z\overline{z})^n}{n!}\frac{\Gamma(\frac{1}{2}+\omega-n)}{\Gamma(\frac{1}{2}+\omega)}\right) \tag{7.9}$$

$$\overline{f}_\omega = \widehat{\mathrm{m}}\ \overline{z}^{\frac{1}{2}-\omega}\sum_{n=0}^{\infty}(\widehat{\mathrm{m}}^2 z\overline{z})^n\frac{1}{n!}\frac{\Gamma(\frac{1}{2}-\omega)}{\Gamma(\frac{3}{2}-\omega+n)} \tag{7.10}$$

One recognizes these functions as short distance expansions of the functions:

$$f_\omega = \Gamma(1/2-\omega)\widehat{\mathrm{m}}^{\omega+\frac{1}{2}}\left(\frac{\overline{z}}{z}\right)^{(\omega+1/2)/2} I_{-\frac{1}{2}-\omega}(\widehat{\mathrm{m}}r) \tag{7.11}$$

$$\overline{f}_\omega = \Gamma(1/2-\omega)\widehat{\mathrm{m}}^{\omega+\frac{1}{2}}\left(\frac{\overline{z}}{z}\right)^{(1/2-\omega)/2} I_{\frac{1}{2}-\omega}(\widehat{\mathrm{m}}r) \tag{7.12}$$

Finally, inserting these expansions into the correlation function, one obtains:

$$\begin{aligned}
&\langle \tfrac{1}{2}|\ \psi_+(r,\varphi)\ \psi_-(r',\varphi')\ |-\tfrac{1}{2}\rangle_{\varphi=\varphi'=0} \\
&\quad = \ \widehat{\mathrm{m}}\pi \left[\left(\sum_{n=1}^{\infty} (-)^n \left(I_{-n-\frac{1}{2}}(\widehat{\mathrm{m}}r) I_{n-\frac{1}{2}}(\widehat{\mathrm{m}}r') - I_{\frac{1}{2}-n}(\widehat{\mathrm{m}}r) I_{\frac{1}{2}+n}(\widehat{\mathrm{m}}r') \right) \right) - I_{\frac{1}{2}}(\widehat{\mathrm{m}}r) I_{\frac{1}{2}}(\widehat{\mathrm{m}}r') \right] \\
&\quad = \ \xrightarrow{\widehat{\mathrm{m}}\to 0} \sqrt{\frac{z}{z'}}\ \frac{1}{z-z'}.
\end{aligned} \tag{7.13}$$

To summarize what we think is interesting, we have shown how from structures in radial quantization we can reconstruct the short distance expansion of a correlation function in an operator framework. It would be very interesting to generalize this to theories other than free fermions, though this seems rather difficult.

Finally we remark that the above formulation of a correlation function is reminiscent of the free fermion construction of tau functions[15]. In fact it is natural to incorporate the dependence on higher coordinates as follows:

$$\tau(z_1, \overline{z}_1, z_2, \overline{z}_2, \ldots..) =< 0|\exp(\sum_n \alpha_n z_n)\Phi(0)\exp(-\sum_n \alpha_n z_n)|\Phi' >$$

The short distance expansion of general sine-Gordon correlation functions is studied in [16]

8. Discussion

Though we have limited ourselves to perhaps the simplest possible case of the free-fermion point of the sine-Gordon theory, we believe the ideas presented here can lead to a new framework for computing form factors in massive integrable quantum field theory. In this approach, since a complete description of the space of fields $\mathcal{H}_F$ is provided from the outset via radial quantization, the complete set of solutions to the form factor bootstrap is automatically yielded. In the bosonized construction in section 5, an important role was played by the affine $\widehat{sl(2)}$ symmetry. Since away from the free-fermion point this symmetry is deformed to a $\mathcal{U}_q(\widehat{sl(2)})$ symmetry[17], this quantum affine symmetry is expected to be important for the general construction. The results contained in [18][19][20] should prove useful.

It is interesting the compare the basic features of our construction with Lukyanov's approach [21] [22], (see also [23]), where the form factors are constructed as traces over

auxiliary Fock-modules. The original motivation behind his construction came from the work[24], where the necessary mathematical properties of these traces were understood in the context of lattice models.

Though the constructions above and [21] are similar in spirit, the detailed aspects of the constructions are quite different; as mentioned above, here the form factors are vacuum expectation values of vertex operators whereas in [21] they are traces over infinite dimensional modules of products of vertex operators. Namely, in [21] the n-particle form factors of a field Φ are represented as

$$Tr_{\mathcal{F}}\left(e^{2\pi i L}V(\theta_1)\cdots V(\theta_n)\,\mathcal{O}_\Phi\right) \tag{8.1}$$

where $\mathcal{F}$ is an infinite dimensional auxiliary Fock space, $V(\theta)$ are vertex operators depending on the rapidity θ, L is the generator of Euclidean rotations (Lorentz boost), and $\mathcal{O}_\Phi$ is an operator that encodes the data of the field Φ.

A completely satisfactory understanding of the origins of these differences is lacking; indeed, the specialization of the results in [21] to the free-fermion point has not been carried out, and furthermore is not an easy exercise. Nevertheless, the main differences in the constructions may be understood heuristically as arising from the distinction between *radial* and *angular* quantization. Let us clarify this point. Define the usual Euclidean light-cone and polar coordinates as follows

$$z = (t+ix)/2 = \frac{r}{2}\exp(i\varphi), \qquad \overline{z} = (t-ix)/2 = \frac{r}{2}\exp(-i\varphi). \tag{8.2}$$

Our work was carried out in radial quantization, where r is declared as the 'time'. In angular quantization, φ is declared as the 'time'. By considering the free-boson limit of the sine-Gordon theory, where an explicit angular quantization may be performed, it was shown in [22] how the main features of the algebraic contruction in [21] arise. In particular, the factor $\exp 2\pi i L$ in the trace was interpreted as a density matrix.

The origin of the traces in angular quantization can also be understood heuristically as follows. In angular quantization, since the Lorentz boost operator L generates shifts in φ, it is the Hamiltonian. Consider now the functional integral formulation of such a quantization scheme. To do this, one must cut the spacetime plane with a semi-infinite line from the origin to infinity. Equal time contours are circles surrounding the origin, where the initial and final times correspond to the two sides of the above semi-infinite line. From the standard correspondence between path integrals and quantum matrix elements,

when one identifies the states on each side of the semi-infinite line and sums over them, one obtains:

$$\langle 0|\ \mathcal{O}\ |0\rangle = \frac{\int\ D\Phi\ e^{-S}\ \mathcal{O}}{\int D\Phi e^{-S}}\ =\ \frac{Tr\ (e^{2\pi i L}\ \mathcal{O})}{Tr\ (e^{2\pi i L})}. \tag{8.3}$$

The $2\pi i$ constant in the factor $\exp(2\pi i L)$ is fixed by the 2π length of the 'time' φ. In [24][21], the latter constant was fixed by imposing the right symmetry properties of the form factors expressed as these traces.

Readers with some familiarity with conformal field theory will doubtless see the strong parallels of this subject with the work presented here. For the example we have developed, we have shown that in radial quantization form factors can be computed as correlation functions in momentum space, and these correlation functions are very similar in structure to conformal spacetime correlation functions. Furthermore, for the purposes of computing form factors, one can describe the space of fields in the same way as is done in the ultraviolet conformal field theory. In a definite sense, we have shown that starting from a description of the space of fields in a conformal field theory and the basic operators from which one constructs this space (in our case, we mean the operators $b^{\pm}_{\omega}, \overline{b}^{\pm}_{\omega}$), then one can reconstruct a massive theory and its form factors by constructing the vertex operators. It is important to understand if this is possible more generally.

As far as correlation functions are concerned, it is disappointing that thus far we have been unable to develop some new methods to derive their main properties from the (quantum) affine symmetry. It would be very interesting to understand even how the differential equations of section 6 are related to the affine $\widehat{sl(2)}$ symmetry.

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